Test of numerical minimization package for the shape optimization of a paper making machine header

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1 Description of the model

We are interested in finding an optimal shape of the back wall of a paper making machine header (see Figure 1). The header is the first component in a paper making machine headbox. Its function is to deliver the fluid (water) and wood fibres equally in the cross direction of a paper making machine in order to produce good quality paper. The cost function to be minimized reads:

$$J(\alpha, \boldsymbol{v}(\alpha), p(\alpha)) := \int_{\Gamma_{\mathrm{out}}} |v_2(\alpha) - v_{opt}|^2,$$

where α is a function describing the shape of the back wall, $(\boldsymbol{v}(\alpha), p(\alpha))$ is the velocity and the pressure of the mixture and v_{opt} is a given target velocity on the outlet Γ_{out}.

The header domain $\Omega(\alpha)$ is of the form

$$\Omega(\alpha) = \Big\{ \boldsymbol{x} = (x_1, x_2) \in \mathbb{R}^2; 0 < x_1 < L, 0 < x_2 < \alpha(x_1) \Big\}.$$

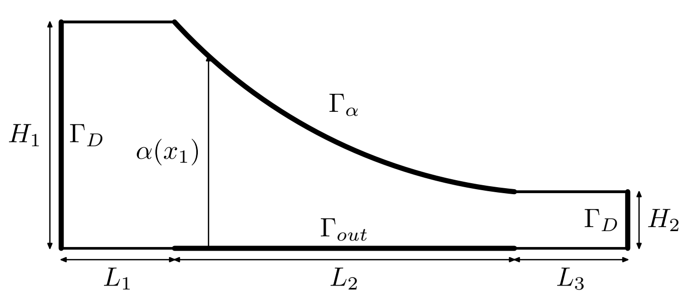

Figure 1: Geometry of the header $\Omega(\alpha)$.

We say that $\Omega(\alpha)$ is an admissible domain iff $\alpha \in \mathcal{U}_{ad}$, where

$$\mathcal{U}_{ad} = \Big\{\alpha \in \mathcal{C}^{0,1}([0,L]);\ \alpha_{min} \leq \alpha \leq \alpha_{max}, \\ \alpha_{|[0,L_1]} = H_1,\ \alpha_{|[L_1+L_2,L]} = H_2, |\alpha'| \leq \gamma \text{ a.e. in } [0,L]\Big\}, \quad (1)$$

and $L := L_1 + L_2 + L_3$.

The motion of the mixture is modelled using the generalized Navier-Stokes system

$$\left.\begin{array}{rcl} -\operatorname{div}\ \mathbb{T}(p,\mathbb{D}(\boldsymbol{v})) + \rho \operatorname{div}(\boldsymbol{v}\otimes\boldsymbol{v}) & = & \mathbf{0} \\ \operatorname{div}\ \boldsymbol{v} & = & 0 \end{array}\right\} \text{in } \Omega(\alpha). \quad (2)$$

Here $\boldsymbol{v} := \boldsymbol{v}(\alpha)$ means the velocity, $p := p(\alpha)$ the pressure, ρ is the density of the fluid and the stress tensor $\mathbb{T}$ is defined by the following formulae:

$$\mathbb{T}(p,\mathbb{D}(\boldsymbol{v})) = -p\mathbb{I} + 2\mu(|\mathbb{D}(\boldsymbol{v})|)\mathbb{D}(\boldsymbol{v}),$$

$$\mu(|\mathbb{D}(\boldsymbol{v})|) := \mu_0 + \mu_t(|\mathbb{D}(\boldsymbol{v})|) = \mu_0 + \rho l_{m,\alpha}^2 |\mathbb{D}(\boldsymbol{v})|,\ \mu_0 > 0,$$

where μ_0 is a constant laminar viscosity and $\mu_t(|\mathbb{D}(\boldsymbol{v})|)$ stands for a turbulent viscosity. The function $l_{m,\alpha}$ represents a mixing length in the algebraic model of turbulence and it has the following form:

$$l_{m,\alpha}(\boldsymbol{x}) = \frac{1}{2}\alpha(x_1)\left(0.14 - 0.08\left(1 - \frac{2d_\alpha(\boldsymbol{x})}{\alpha(x_1)}\right)^2 - 0.06\left(1 - \frac{2d_\alpha(\boldsymbol{x})}{\alpha(x_1)}\right)^4\right),$$

where $d_\alpha(\boldsymbol{x}) = \min\{x_2, \alpha(x_1) - x_2\}, \boldsymbol{x} \in \Omega(\alpha)$. The following boundary conditions are assumed:

$$\begin{array}{rcll} \boldsymbol{v} & = & \mathbf{0} & \text{on } \Gamma_f \cup \Gamma_\alpha, \\ \boldsymbol{v} & = & \boldsymbol{v}_D & \text{on } \Gamma_D, \\ \boldsymbol{v}\cdot\boldsymbol{\tau} = v_1 & = & 0 & \text{on } \Gamma_{out}, \\ T_{22} := \mathbb{T}\boldsymbol{\nu}\cdot\boldsymbol{\nu} & = & -\sigma|v_2|v_2 & \text{on } \Gamma_{out}, \end{array} \quad (3)$$

where $\boldsymbol{\nu}, \boldsymbol{\tau}$ stands for the unit normal, tangential vector to Γ_{out}, respectively and $\sigma > 0$ is a given suction coefficient. The condition $(3)_4$ originates in the homogenization of a complex geometry.

2 Approximation and test results

The state problem (2) is discretized using the finite element method on a triangular mesh. In order to compute the gradient of J with respect to α, the adjoint equation technique is applied to the discrete state problem.

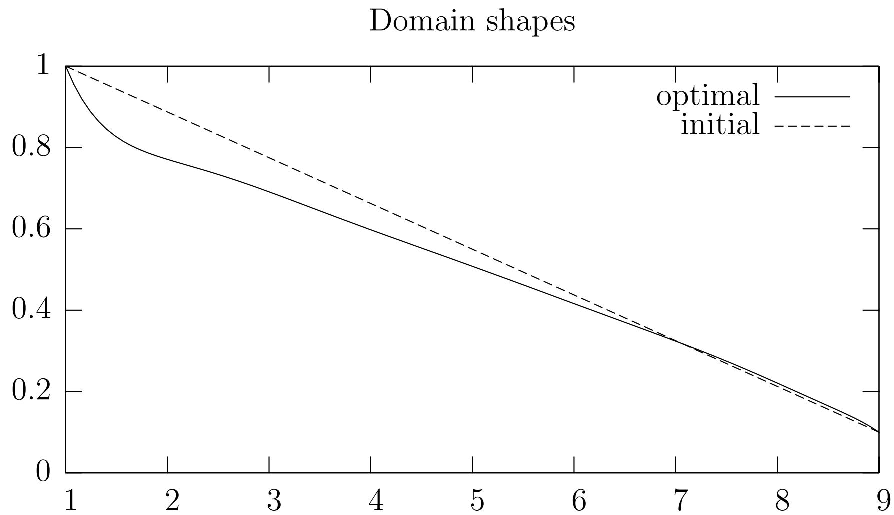

Figure 2: Initial and optimal shape.

All necessary partial derivatives are provided with help of the automatic differentiation. The function α is approximated by a Bézier function α_M of order M. Admissibility of α_M is imposed by simple lower and upper bounds only.

Finally, our problem can be formulated as a nonlinear, bounds-constrained programming problem for which the cost function value and gradient are available. We used the **NAG** C library, routine e04wdc [1] to solve this problem, using the default parameter values.

In the test computation we used the following dimensionless parameters: $L_1 = 1.0$, $L_2 = 8.0$, $L_3 = 0.5$, $H_1 = 1.0$, $H_2 = 0.1$, $\alpha_{min} = H_2$, $\alpha_{max} = H_1$, $\mu_0 = 10^{-3}$, $\rho = 10^3$, $\sigma = 10^3$, the inlet velocity $\boldsymbol{v}_{D|\{0\}\times(0,H_1)} = (4(1-(\frac{2x_2}{H_1}-1)^8), 0)$, $\boldsymbol{v}_{D|\{L\}\times(0,H_2)} = (1-(\frac{2x_2}{H_2}-1)^8, 0)$. The order of the Bézier function α_M was set $M = 20$, and the target velocity $v_{opt} := -0.433$.

The optimization started from the traditional linearly tapering shape. The initial and optimal shape (found by e04wdc) is depicted in Figure 2 and the initial and optimal velocity profile $v_{2|\Gamma_{\mathrm{out}}}$ is depicted in Figure 3.

The NAG routine found the solution after 73 major iterations, yielding the optimality error smaller than 10^{-6}.

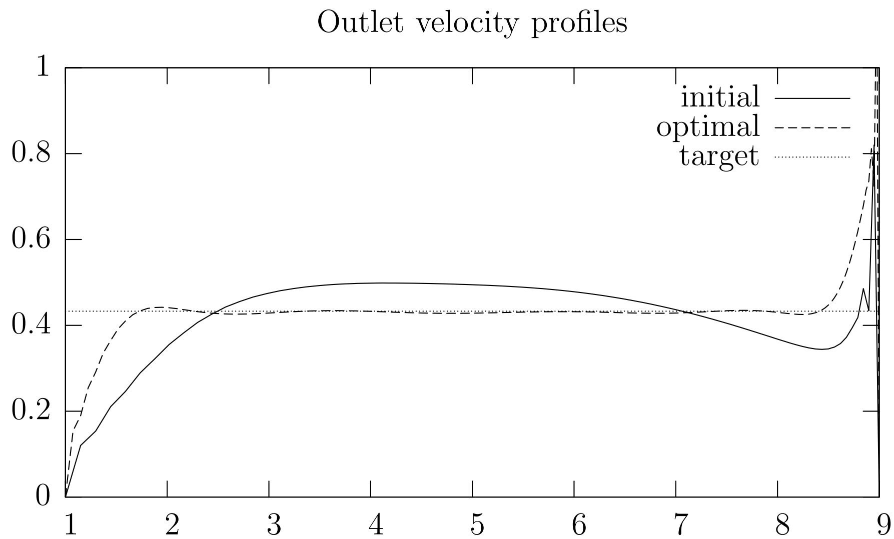

Figure 3: Initial and optimal velocity profile.

3 Conclusion

The NAG optimization routine shows itself to fully meet the demands of the problem.

References

[1] P. E. Gill, W. Murray, and M. A. Saunders. *SNOPT: An SQP Algorithm for Large-scale Constrained Optimization*. SIAM J. Optim., 12:979-1006, 2002.